Section 1. Mathematics and Science

DOING MATH WITH MOTION SENSORS

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Abstract: *The concept of function is one main concept to teach interdisciplinary in mathematics and science. Using a motion sensor to explore common aspects in mathematics and physics can enhance mathematical and physical learning in upper secondary level. The potential for deeper learning and the opportunities to teach these concepts interdisciplinary are discussed.*

Motivation

Everybody having a parking-assist-system in his car, gets involved with motion sensors. They measure the distance between the car and the nearest object to it. If the distance comes below a certain distance, an acoustic signal will be activated. Bigger versions of such a motion sensor are used in physics class to measure the distance of an object to this device at a certain time. The measuring values can be plotted as distance vs. time graphs. Students can see the results of that measurement in real-time. This simple experiment has a great potential to explore different mathematical and physical concepts. Furthermore this experiment can be used to teach these concepts interdisciplinary.

Mathematical and physical concepts involved

When students examine distance vs. time graphs different concepts can be touched. From a mathematical view these are the concept of function, the average rate of change, the instantaneous rate of change following the concept of the derivative. From a physical view several competencies can be touched: students describe movements of objects verbally, they determine average and instantaneous velocity and sketch velocity vs. time graphs, when distance vs. time graph is given. If one compares these concepts respectively competencies analogies can be found. That's why this could be one opportunity to teach interdisciplinary involving math and physics. Interdisciplinary teaching should only be done, if that form of teaching will enrich learning of concepts and methods more than conventional lessons would achieve that (Beckmann (2003), p.30). This means that interdisciplinary teaching should lead to a more deepen understanding of concepts and methods. Considering the concept of function this involves at least understanding the aspect of correspondence and the aspect of covariance (Hoefer (2008), p.52ff). Aspect of correspondence means that each argument corresponds to only one value. The aspect of covariance considers the changing behaviour of a function, i.e. changing the argument (usually) changes the function value. Obviously both aspects can be touched by the use of a motion sensor to examine the concept of function and the concept of derivative as well. Another didactical aspect of the concept of function involves changing the forms of representation. Using a motion sensor the graph of a function is the most dominant form of representation. The graph is the initial form to start with. How these changes of forms of representations can be realised in class will be shown later.
Considering the competencies for physics class, one can see that these can also be touched and learned at the same time. For students this isn't always that obvious. One obstacle are

different notations for the same objects which is illustrated in table 1. This obstacle must be known to mathematics teachers, especially for teachers not teaching physics. If students can see, that in both subjects they deal with the same thing, it will be easier for them to think interdisciplinary.

Mathematics	Physics
Position vs. time graph	Position vs. time diagram
Average rate of change, Slope of a secant	Average velocity
Instantaneous rate of change, Slope of a tangent	Momentary velocity
Graph of the derivative function	Velocity vs. time diagram

Table 1. Different subject – different name – same object

Realisation in class

At the beginning a few movements will be recorded. By looking at the graphs the students describe the movements without using formal mathematics. Students explore that the steepness of a curve tells them, how fast a person moves. Usually this is written down by sentences like "The steeper the curve is, the faster the person moves." One can tell in which direction a person moves if the curve rises respectively falls. This activity can be realised in mathematics or physics class. Especially weaker students, who have problems with formal mathematics, can make contributions in such a lesson. These results can be used for the following lessons in both subjects. In mathematics the concept of function as unique correspondence will be emphasized first followed by deeper analysis of the changing behaviour of a curve. By using the movement graphs the concept of function and rate of change will be made explicit. The same graphs can be used in physics class. To find the average velocity during a certain time interval, one has to look at the changing behaviour of a curve. The formula of the average velocity $\bar{v} = \frac{s_1 - s_0}{t_1 - t_0}$ corresponds to the average rate of change of the distance vs. time function. This can lead to a discussion, that a position vs. time function can be written as s(t) and f(x) as well. They mathematically describe the same object, but following physical convention it is written as s(t). To emphasize the common structure of average rate of change and average velocity the formula above can be modified to
$\bar{v} = \frac{s(t_1) - s(t_0)}{t_1 - t_0}$. By looking at this formula one can also analyse analogies between both subjects. If one draws the graph of a secant it will represent a movement with constant velocity. The activities to determine the average velocity in physics class correspond to the activities to determine the slope of a secant. In both subjects the slope of a secant can be determined by a graph or by a table of measuring values. One aim of interdisciplinary teaching involves exploring common structure of different subjects and limits of disciplinary views (Zell (2010), p. 33). This aim can be reached by analysing in which intervals it makes sense to determine the average velocity.

The movement illustrated in Figure 1 shows that it might not be useful to determine the average velocity for the whole movement. One can also see that there are intervals in which the average velocity is about the same like the actual velocity. So the average velocity can be one tool to determine the actual velocity or mathematically spoken the slope of a secant can be one tool to determine the rate of change of a function. Since this is not true for all intervals, one needs a method to determine the rate of change for all intervals. That leads to the concept of instantaneous rate of change respectively momentary velocity. In both subjects the problem can be solved by dealing with the differential quotient. But now the intention using the differential quotient in each subject differs. In mathematics lessons the main intention is a formula for the differential quotient und introducing rules to find the derivative of a given function. This finally leads to determine local maxima resp. minima, points of inflection and so on.

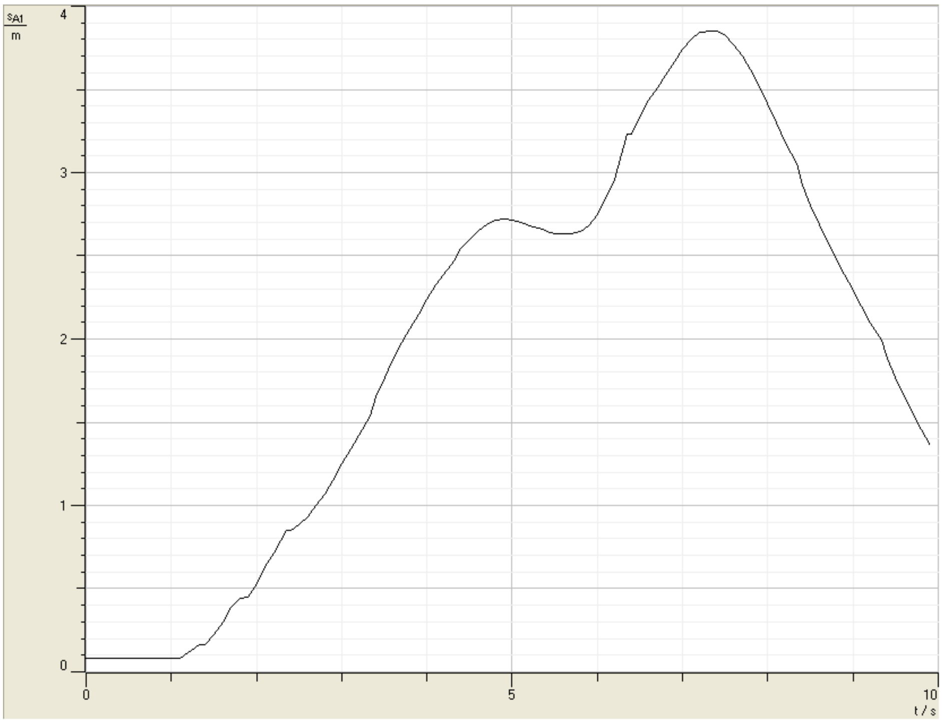

Figure 1. Movement of a person recorded by a motion sensor

In physics lesson the momentary velocity will be determined out of a given graph (most of the time) since many real movements cannot be described by an easy equation. In the lessons afterwards the main objective will be movements with constant velocity or constant acceleration. Since there are different intentions in both subjects one can see that interdisciplinary teaching would rather interfere an enriched learning of concepts or methods. Other aims of interdisciplinary, teaching holistic approach and exploring common structures of subjects involved, can be reached rather slightly. At that moment it shouldn't be taught interdisciplinary. A few weeks later the same movement graphs can be used in mathematics lesson. To motivate the introduction of the derivative function the teacher can ask students to determine the (momentary) velocity of a person for the whole interval. The graph of a derivative function can be described by both slopes of a tangents and momentary velocities. By introducing the derivative function like that, students will explore that concept in a less abstract way. This way especially helps weaker students who have problems in abstract thinking. The same activity is usually done in physics class when students have to draw a velocity vs. time graph with a position vs. time graph given. Since the acceleration is the derivative function of the velocity, analogies can be shown in math and physics class.

The concepts mentioned are central parts in the respective subjects and the respective school curricula. They are the foundation for concepts following. Therefore a profound understanding is necessary and can be reached in an interdisciplinary way.

Thoughts about interdisciplinary teaching

This interdisciplinary approach at the beginning of upper secondary class shows how mathematical and physical concepts and competencies can be reached in a meaningful and less abstract way. The profoundness of understanding, subject-specific and interdisciplinary, depends on the way how interdisciplinary teaching is organized (Beckmann (2003), p.8ff). Interdisciplinary teaching can be realised in an ordinary math lesson. In that lesson the teacher goes beyond the boundaries of his subject, at least for a while, and uses aspects of another subject to enhance mathematical learning. Interdisciplinary teaching can also be realised if the same topic is taught in mathematics class and in another class, i.e. physics class, at the same time. The most intense organization forms of interdisciplinary teaching can be realized, if teachers plan their lessons in common and coordinate the order and contents of their lessons or they overcome their subject boundaries and teach one topic in an inter-subject way (cf. Beckmann (2009), p.8ff). Different organizational forms are possible in the lessons presented above. The motion sensor experiment could be carried out in mathematics class and mathematical aspects (and slightly physical aspects, depending on the teacher) can be touched and learned by the recorded movement graphs. It is also possible that a mathematics teacher and a physics teacher teach this topic at the same time and work with analogue movement graphs. If teacher coordinate their lessons commonalities between both subjects will be even more obvious. The best way to see commonalities would be working with the *same* movement graphs in mathematics and physics lessons. Then it would be easier for students to find analogies between mathematics and physics on their own.

Experiences in school

The use of a motion sensor in school has been tested in school throughout different school years. In one year the motion sensor has been used in mathematics class only. There different motions have been examined to make connections between the graphs and the real movements. Then the slope of a secant has been worked out of these movement graphs followed by the slope of a tangent. The concept of average resp. instant velocity has been touched slightly during the introduction into the concept of secant and tangent. In the same way the use of a motion sensor has been tested in a physics class only. The first lesson was identical to the one in mathematics class. After that lesson the concept of average and instantaneous velocity has been introduced. The concept of secant and tangent has been touched only slightly by comparing the formulas given in physics and mathematics class. In both, mathematics and physics, classes especially weaker students profited by the use of the motion sensor in class. The concepts taught were more obvious to them, since they connected these concepts to certain types of motion. In another school year math and physics were taught in a more interdisciplinary way. Mathematics and physics lesson were coordinated before the lessons started. Some lessons had been changed within the schedule. The commonalities between both subjects could have been shown more intensively and students profited even more. But that coordination was made possible easily since both subjects were taught by the same teacher. Coordination between teachers is more common the smaller a school is (Zell, 2010, p79f). So one aim of further research is to find ways for better coordination among teachers in bigger schools, another aim could testing an extended use of the motion sensor in mathematics class, since the use of that device supports especially weaker students. This could be realised and tested for the concept of function in early secondary class and the concept of integral in upper secondary class.

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MATHEMATICAL MODELING – THE DIDACTICAL LINK BETWEEN MATHEMATICS AND BIOLOGY

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Abstract: *The absence of strong curricular ties between biology and mathematics misrepresents contemporary biological research, and the need for more mathematics in biology education should therefore be addressed. The method of mathematical modeling applies very broadly in many biological fields including some like population growth and spread of disease. This paper focus on how to strengthen the educational relations between mathematics and biology by interdisciplinary teaching centred on modeling activities. As background interdisciplinary aspects of the scientific practice of mathematics and biology is shortly addressed. The main part of the paper consists of presentations of modeling as a core element in a framework for interdisciplinary teaching and two illustrative examples of interdisciplinary teaching sequences encompassing mathematics and biology developed by teacher-students. It is concluded, that modeling activities provides a generic methodology that serves as a common denominator for learning disciplines, such as mathematics, biology – and chemistry and physics.*

Introduction

The Danish upper secondary education is organized in specialized studies packages containing compulsory subjects, core subjects, and elective subjects. An important feature of a package is that the core subjects form a coherent program, which is ensured by a closer interaction between the subjects. Some of the packages include mathematics and biology as core subjects. To fulfil the objective of coherence in the subject packages interdisciplinary teaching across mathematics and biology is demanded. However, although many major parts of biology, e.g. ecology and evolution, have benefitted from mathematics, there are no strong curricular ties between mathematics and biology at upper secondary education. This makes the demand of coherence in the subject packages a challenging task for teachers, and calls for a didactical framework for interdisciplinary teaching and prototypes of meaningful interdisciplinary teaching sequences between mathematics and biology.

This paper focus on how to strengthen the educational relations between mathematics and biology by interdisciplinary teaching centred on modeling activities. As a background, the interdisciplinary aspect of the scientific practice of mathematics and biology is shortly addressed. The main part of the paper consists of presentations of modeling as a core element in a framework for interdisciplinary teaching and illustrative examples of interdisciplinary teaching sequences encompassing mathematics and biology developed by teacher-students.

Mathematics and biology

Most science and mathematics educators are familiar with the famous article by Nobel Laureate Eugene Wigner on the philosophy of mathematics and of science: 'The Unreasonable Effectiveness of Mathematics in the Natural sciences'. Wigner (1960) describes the usefulness of mathematics in natural science sciences as bordering on the mysterious with accuracy beyond all reasonable expectations and with no rational explanation for it. Scarcely as famous as the description by Wigner is the comment by the famous Russian mathematician Israel Gelfand, that there is only one thing, which is more unreasonable than the unreasonable effectiveness of mathematics in physics, and this is the unreasonable ineffectiveness of mathematics in biology (Borovik 2010). However, this conception of the role of mathematics in biology is challenged by the historical and actual practice of biology and mathematics. Through the times, mathematics has been inspired by biological problems, and by this, mathematical concepts are produced and becoming central elements of the culture of mathematics. An example is the Fibonacci numbers appearing in the pedigrees of idealized honeybees. This is one of the first examples of a population model resulting in exponential growth, and on top of that with a golden growth rate. The immediate effect of Fibonacci's work was not the study of living organisms, and the Fibonacci sequence continues to delight and thrill mathematicians. Moreover, mathematicians' considerations in the field of biology have shaken the world. Surely, one of the most recognized examples is Thomas Malthus' warning about the calamitous consequences of population growth. Malthus argued that population multiplies geometrically and food arithmetically; therefore, the population will eventually outstrip the food supply (Malthus 1798). From Malthus warnings in the 19th century there is a clear connection to the resource constraints and biological consequences foreseen by the Club of Rome's famous book, 'The Limits of Growth', which rely heavily on mathematical models (Meadows et.al. 1972).

Also, some of the most prominent members of the international mathematical society challenge Gelfand's theme. To celebrate the World Mathematical Year 2000 the American Mathematical Society under the auspices of the International Mathematical Union published the book 'Mathematics: Frontiers and Perspectives'. In the preface of the book the world famous mathematician Michael Atiyah reflects over mathematics in the 21st century and predicts that real-world problems will have profound impact on the development of mathematics in the 21st century. Mathematics is affected by other sciences, and according to Atiyah it is not unreasonable to think that mathematics may have a useful role part to play in understanding the brain and handling the vast bank produced by the Human Genome Project (Atiyah 1999). The widely known popular-math writer Professor of Mathematics Ian Stewart argues in his book 'Life's other secret', that if we keep discovering ways in which mathematics informs biology, we will discover the deep structure of life. The patterns of nature are made from mathematical ingredients, and mathematical patterns provide building blocks and a place from which genetics and evolution can start (Stewart 1998).

The last century in the history of mathematics is characterized by the increasing influence of applied mathematics. In such different fields as engineering, economics, biology, and medicine applied mathematics has played, and still plays a more and more important role in new development and breakthroughs (Steen 2005). Mathematicians, statisticians, and biologists have strived for a century to develop effective models in biology. Jungck (1997) presents ten mathematical models equations that changed biology and point to that the exceptionally roles played by mathematics throughout the history of biology is unappreciated in biology curricula. According to Jungck (1997, 2005) this absence of strong curricular ties between biology and mathematics misrepresents contemporary biological research, and the need for a more mathematics in biology education and problem-solving based curriculum in biology should therefore be addressed. The emergence of models and the existence of large data sets that require quantitative analysis present a great opportunity for the mathematical sciences. Moreover, one could add, a great opportunity for mathematics education too. The foundations of many fields of biology and in particular the new fields are inherently mathematical. The method of mathematical modeling applies very broadly in many biological fields including some like population growth and spread of disease.

A didactical framework for interdisciplinary teaching

Although there is a need for a curricular change from mono-curricular to cross-curricular aimed at preparing students for the emerging interdisciplinary fields that transcends the boundaries of technology, biology and mathematics, there is still no broadly accepted framework provided for an interdisciplinary approach. It is crucial that such a framework is provided; else interdisciplinary intentions will end up in a subjects-based perspective, in which any links predominantly are left to take place in the students' minds. In the following, we propose a didactical framework interdisciplinary teaching made up of two pillars:

- The conception of modeling as an interdisciplinary teaching and learning activity
- A didactical model providing a structure for interdisciplinary activities

Modeling as an interdisciplinary activity

Instead of focusing on how to overcome the challenges of implementing mathematics into biology Jungck (2011) suggest to the development of individual biological models that can be easily adopted and adapted for use in both mathematics and biology classrooms. The extensive literature recognizing the importance of models and modeling, both in mathematics education (Blum, Galbraith, Henn & Niss 2007) and in science education (Gilbert 2004), indicates that modeling might provide a generic methodology that can serve as a common denominator for learning subjects such as biology and mathematics - and chemistry and physics as well. Despite the overwhelming amount of literature on modeling in science and mathematics education, the interdisciplinary potential of modeling is seldom addressed explicitly. However, modeling is a specific problem solving strategy with scientific, mathematical and pragmatic purposes (Hesteness 2008). In science education it is often accentuated that many phenomena and their patterns of interaction are best described in the language of mathematics, which then becomes a bridge between the students' verbal language and the scientific meaning we seek to express (Osborne 2002).

In Denmark the notion of mathematical and science competences function as a basis for describing and analyzing mathematics and science teaching from kindergarten to tertiary level. Eight mathematical and four science competences are identified, and the competency of modeling is identified both as a mathematical and science competence. In mathematics the modeling competence includes structuring an intra- or extra-mathematical situation to be modelled, mathematizing the situation, analyzing and tackling the model, interpreting the results, validation of the model, communicating about the model, monitoring the modeling activity (Niss & Højgaard 2011). The reference to the modelling of extra-mathematical situation underscores that the competence should not be considered as a subject specific, but as an interdisciplinary competence (Michelsen 2006). Thus the interdisciplinary competence of modelling is the glue that holds together the interacting subjects.

The didactical model for interdisciplinary teaching

According to Freudenthal (1991) mathematizing is the key process of the mathematical practice. There are two types of mathematization in an educational context – horizontal and vertical. In horizontal mathematization, the students come up with mathematical tools, which can help to organize and solve a problem located in a real-life situation. Vertical mathematization is the process of reorganization within the mathematical system itself, like finding shortcuts and discovering connections between concepts and strategies, and the application of these discoveries. The process of horizontal mathematization is strongly connected to situation in which a problem, e.g. a biological problem, is translated to a mathematical model. The process of vertical mathematization then anchors the mathematical model, e.g. exponential growth, within a mathematical conceptual framework and makes it possible to transfer the model from one place to another. This makes Freudenthal's approach a promising theoretical framework for designing instructional sequences to strengthen the relations between mathematics and the extra-mathematical world, e.g. biology.

Michelsen (2006) proposes an extension of the distinction between horizontal and vertical mathematization to a didactical model for integrated mathematics and science courses. The model consists of two phases: horizontal linking and vertical structuring. The description of the modelling competence as an interdisciplinary competence entails that situations from biology are embedded in the contexts to be mathematized, and this takes place is the horizontal linking phase. The vertical structuring phase is characterized by a conceptual anchoring of the concepts and process skills brought into play by the students in the horizontal linking phase by creating languages and symbol systems that allow the students to move about logically and analytically within mathematics and biology without reference back into the contextual phase. To illustrate the application of the two phases of the didactical model, we look at consumption and elimination of alcohol. In the horizontal linking phase the focus is on the thumb rule saying that a person will eliminate one average drink of alcohol per hour. The problem the students have to deal with is to mathematize the thumb rule, and construct a mathematical model for the rate of metabolizing alcohol. In the vertical structuring phase the model constructed in the horizontal linking phase acts as a starting point for the students to investigate and conceptualize concepts like metabolism, diffusion and concentration in biology and linear growth models, variables, rate and parameters in mathematics.

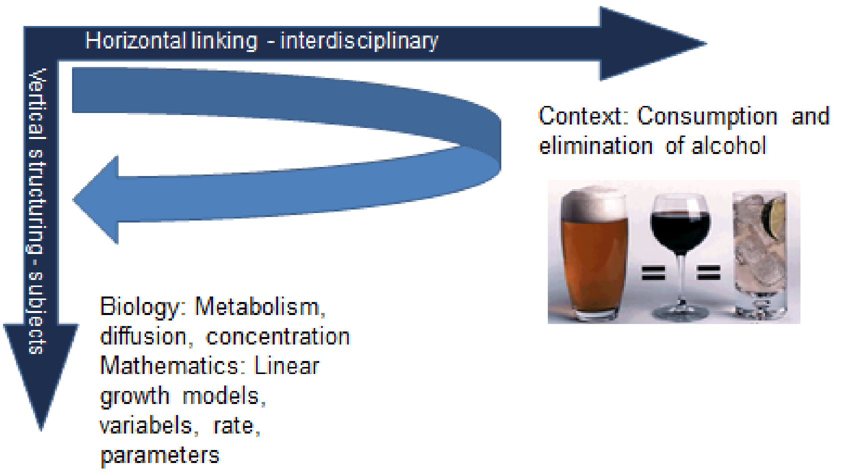

Figure 1. The didactical model of horizontal linking and vertical structuring

The shift from the horizontal to the vertical phase might thus concur with a shift from integrated teaching to subject-oriented teaching. It should be stressed that the framework is iterative. Once the concepts and skills are conceptually anchored in the respective subjects, they can evolve in a new interdisciplinary context, as part of a horizontal linkage.

The Modeling course for pre-service teachers

In an attempt to offer future science and mathematics teachers the possibility to prepare themselves for the practical challenges of teaching interdisciplinary, the University of Southern Denmark offers since the academic year 2008/2009 a graduate course in ‘Modeling and interdisciplinary teaching’. The course title explicitly highlights that the main idea of the course is modeling as a tool for interdisciplinary teaching. The course ‘modeling and interdisciplinary teaching’ is based on the didactical framework outlined above, a syllabus of research literature concerning interdisciplinarity, and activities involving the students in developing sequences of interdisciplinary teaching. For the implementation of the course, the students are to do a final 15-20 page exam project, where they design an interdisciplinary teaching module and clearly (i) describe the form and extent of the interdisciplinarity, and the place and role of modeling, and (ii) argue for their design choices on the basis of the course literature. Furthermore, the students are asked to structure the project around an actual problem formulation. In the following, we provide two examples of student projects in order to, on the one hand, to give concrete examples of interdisciplinary modeling activities encompassing the subjects of mathematics and biology, on the other hand, to illustrate the type of problems regarding interdisciplinary teaching that the students themselves came to realize as being imperative for the success of designing and implementing interdisciplinary teaching activities at upper secondary level[4].

Two examples of student projects

In the first example, the students outline a teaching module entitled ‘From insects to human metabolism and physical fitness’. According to the students the inclusion of mathematical modeling in an interdisciplinary teaching module has the potential to bridge the gap between the ‘world of biological objects’ and the ‘world of mathematical formulas’. They state:

> In our view, it is a general problem that many students have difficulty applying what they have learned in mathematics in other disciplines, including biology. On the other hand, in mathematics lessons they find it hard to see the usefulness of what they are doing which for some students can be very demotivating. The starting point of this interdisciplinary module has therefore been to meet both of these issues with a focus on modeling - a general competence, which is extremely important, both in biology and mathematics. (Sørensen, Oechsler-Christensen & Kristensen 2014, p. 15)

The module addresses the topics physiology and linear relationships, and both topics are part of the core content of biology and mathematics, respectively. According to the students the chosen topics provide an interdisciplinary context for horizontal linking composed of a classic biology experiment about determination of physical fitness and the handling the data of the experiment by mathematical modeling:

> The experiment and the subsequent data processing link the content, the problems and the pupils’ actions horizontally, and hopefully the pupils experience how much the two subjects have in common (Michelsen 2005). Depending on the pupils’ performance it may be fruitful to increase the complexity of the experiment by repeating it at different temperatures, which produces linear models with different slopes. This may

[4] In the following the quotations from the exam reports are translated to Danish by the author, and ‘students’ refers to the student-teachers enrolled at the course, while ‘pupils’ refers to the intended target group of the designed modules; upper secondary school pupils.

> help to increase the pupils' understanding of linear models. (Sørensen, Oechsler-Christensen & Kristensen 2014, p. 17)

The students address the issue of promoting the general modeling competencies of the pupils:

> Throughout the design process of this interdisciplinary teaching module, it has been our intention to strengthen students' general modeling competencies through the interaction between biology and mathematics. We have therefore chosen to design the module so that pupils prospectively can improve their skills in applying and constructing models. (Sørensen, Oechsler-Christensen & Kristensen 2014, p. 22)

The students also discuss the challenge of introducing the modeling approach to pupils that are not familiar with this approach:

> The majority of the pupils are not familiar with the modeling approach, and this may place considerable demands on their level of abstraction. We are fully aware that all students will achieve the full understanding of the models. (Sørensen, Oechsler-Christensen & Kristensen 2013, p. 20)

In the second example, the students outline a teaching module entitled 'Production of bioethanol' incorporating biology, chemistry, and mathematics. The rationale for choosing this theme is that the students expect it can act an 'appetizer' for engaging and motivating the pupils for interdisciplinary activities. The students structure their exam report around the following problem formulation:

> How should we organize a course on the topic production of bioethanol as an interdisciplinary activity including the subjects of chemistry, biology and mathematics? How can interdisciplinary modeling increase the students' interest in and commitment to science and mathematics? (Christensen & Dall 2013, p. 5)

The modeling activity of the module is the construction of a mathematical model for the relations between temperature and reaction rate of two key processes in industrial manufacturing of bioethanol; degradation of cellulose to glucose and fermentation of glucose. Based on their models the pupils are asked to assess if it is worthwhile to produce bioethanol. The students accentuate that theme production of bioethanol and the modelling activities have a societal dimension:

> The strength of working with modelling with the aim of estimating the optimal temperatures is that the pupils experience how scientific modeling is applied in real world settings. This fulfills the objective of science and mathematics education at Danish upper secondary school, that the pupils should be able to put science into a cultural perspective and discuss the application of science in society. (Christensen & Dall 2013, p. 35)

Like it was the case in the first example students addresses the issue of general modeling skills:

> If we were asked to on basis of the experience gained in this exam project to design a new interdisciplinary module on modeling, we would consider a radical different approach. Instead of involving the pupils in modeling activities parallel to the teaching of the core content of the involved subjects, we would design a short introductory module entitled 'Modeling'. The module should not only address construction and application

> of models, but also offer an introduction to the concept of modeling and types of models and their role in science. (Christensen & Dall 2013, p. 36)

Finally the students point out that planning interdisciplinary teaching puts demands on the teachers:

> Traditional subject boundaries (..) require a high degree of planning, motivation and flexibility from teachers to get the different subjects within the interdisciplinary module to play sensible together and thereby independently illuminate the problem at hand (Christensen & Dall 2013, p. 5).

Concluding remarks

The problem addressed in this paper is twofold. On one hand, mathematics evidently has played and will play an exceptionally important role in the development of biology, but this role is underrepresented in biology curricula. On the other hand, mathematics learned at upper secondary level seems to have little relevance to the biology taught. With this paper we have tried to show that modeling provides a generic methodology for learning disciplines, such as mathematics and biology, and that not only can mathematics support the understanding biological phenomena, but also biological knowledge can act as a background for understanding mathematical concepts.

The two examples of interdisciplinary modules from the students' exam projects show that the future teachers are able to articulate interdisciplinary themes capturing the relationship between the subjects of mathematics and biology. Especially the students relate mathematical modelling to experiments by including in their teaching modules the iterative process of (i) experiment and data, (ii) mathematical model, (iii) parameter estimation, and (iv) simulation. The didactical framework provides the students with a structure for identifying interdisciplinary themes with a significant content for the participating subjects, and modeling serves as the unifying activity in the students' modules. This indicates that although the course has a short history, and we are to truly see the outcome of the course when the participants have begun teaching at upper secondary level, exploration of the pedagogical potential of modeling should be considered critical for the professional development of mathematics and biology teachers. Also it is worth noticing that in both exam report the issue of general modeling skills is addressed. Doubtless, the success of modeling as a tool to strengthen the interplay between mathematics and biology - and physics and chemistry as well - is strongly dependent on a clarification of modelling as an interdisciplinary competence and identification meta-modeling competencies.

From an research point of view, it is remarkable that in spite of the comprehensive research in modeling in mathematics education as well in science education only little attention has been paid to research on the role of modelling in the educational relations between mathematics and biology. To fulfil the educational desire to integrate mathematics and biology education, interdisciplinary teaching should be sustainably integrated in teachers' practice, and based on educational research in interdisciplinary aspects of mathematics and biology. Accentuating interdisciplinary teaching and modelling between mathematics and biology also entails taking a stance to the challenge of involving technology and engineering in teaching. For example, bioinformatics rely on both mathematical modelling and digital instrumentation. Bybee (2013) argues that the so-called STEM[5]-approach is needed as an educational position that

[5] STEM is an acronym for Science, Technology, Engineering, Mathematics

first place life situations and global issues in a central educational position and then uses the four disciplines of STEM to understand and address the problem. Thus competency in addressing based situations, problems, or issues is emphasized, and this calls for an extension of the didactical model outlined above to include engineering to improve students' understanding of how things work and improve their use of technologies.

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