Section 3. Mathematics and Arts

EXPLORING DEEPER CONNECTIONS BETWEEN MATHEMATIC, MUSIC AND ITC: WHAT AVENUES FOR RESEARCH IN EDUCATION?

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Abstract: *Music and mathematics share a rich common heritage which is not being fully explored in educational contexts. This paper attempts to further extend the discussion about possible gains that could be obtained in making more explicit connections between the two disciplines in our classrooms. We want to outline that being aware of the links between mathematics and music through creativity is a sure way to learn and understand one and the other, and maybe to learn and understand others fields too. More specifically, this paper focus on possible links related to creativity and ICT (Information and Communication Technology). By looking into the mathematical patterns emerging in children's musical creations, we expect that new links between mathematics and music can be recreated through interdisciplinary teaching with ICT by focusing on creativity.*

1. Introduction

Music and mathematics share a rich common heritage which is not being fully explored in educational contexts. Henle (1996), for instance, compared the history of music with the history of mathematics, asserting that: "Mathematics has many of the characteristics of an art, viewed as an art, it is possible to identify artistic periods in mathematics: Renaissance, Baroque, Classical and Romantic, and that these periods coincide nicely and share many characteristics with the corresponding musical epochs, but are significantly different from those of painting and literature." (p. 19). He draws out similarities between the evolution of mathematics and music and outlines the necessity of a change in mathematical education towards a more musical style. "Students should make mathematics together (…), not alone. (. . .). And finally, students should perform mathematics; they should sing mathematics and dance mathematics" (p. 28).
In this paper, we will discuss some of the possible gains that could be obtained in making more explicit connections between the two disciplines in our classrooms. More specifically, we will focus on possible links related to creativity and ICT (Information and Communication Technology). The origins of our questioning arise from the study which one of the authors, Xavier Robichaud, is conducting within his Ph. D. project. When analyzing elementary students' musical creations with the use of iPads, we are intrigued by the richness and diversity of patterns of mathematical nature in their work, as well as by the explanations given in the interviews.

After a review of the history shared by mathematics and music in a variety of time periods and contexts including educational ones, we discuss examples of students' work analyzed within a cultural framework of creativity (Glavanu, 2014).

2. Historical background

From ancient times, music was closely associated to mathematics. In the sixth century BC, for instance, Pythagoras argues in his famous experiment of ropes of different lengths, that the notes of the scale can be considered as mathematical ratios (the intervals relationship perceived as pleasant can all be expressed through numbers 1, 2, 3, 4). Whether Euclid (I c. BC), with is theory of musical intervals, Boethius (VI c.), whose music theory calculates intervals with the monochord or Galileo (XVI c.), who studied the musical intervals and showed that the pitch of the sound originate from vibration frequency; all of them used mathematics to analyze the phenomenon of music. In the seventeenth century, several renowned mathematicians like Kepler, Wallis, Mersenne, Desargue or Descartes also wrote on music (Archibald, 1924).

Designed and based on mathematically harmonious numerical relationships, the music of our civilization was seen as an emblem of "world harmony". Leonhard Euler (XVIII c.) was one of such mathematicians who represented this ideology. He based his argument on the assumption that human beings feel pleasure if they can perceive perfection. And this perfection comes down to the perception of some sort of order. In music, the order comes from the relationship between the sounds heard. Each sound may be represented by a number; musical chords thus become relations of numbers. But this view is not unanimous; Aristoxenus (IV c. BC) asserts that music is to be judged by the auditory sensation (Beaugé, 2002). D'Alembert (XVIII c.) openly criticized Euler by arguing that music theory is a matter of natural sciences (relevant to auditory sensations) and not of pure mathematics (Paquier, 2008). This is the school of thought that prevailed since Helmholtz (1868) with his physiological theory of music based on auditory perception. However, we must not forget that the mathematical ideology still persists and innovates: Schoenberg (1874-1951) invented a system that has had a major influence on twentieth century music. He named it "Reihenkomposition": every piece of music should be based on a "series" of twelve sounds that could be followed in any order, with the restriction that the same sound should not be repeated twice (Leibowitz, 1947).

In the twentieth century, the advent of computer technology opened new horizons, including the application of new technologies in music and mathematics field (AFIM, 2009; Bamberger J. & DiSessa A, 2004).

3. Connections between mathematics and music related to creativity and technology

There are works which bring together mathematicians and musicians. While looking into possible links between mathematics and music, mathematicians and musicians use different perspectives: mathematicians are often studying mathematical structures as the base of musical work; music researchers seek for connections between musical and mathematical skills, usually to claim that, at the school level, it is beneficial to be a musician as a predisposition to be good in mathematics. For instance, several authors argue that music targets one specific area of the brain to stimulate the use of spatial-temporal reasoning, which is useful in mathematical thinking (Vaughn, 2000; Rauscher, 2006, Zhan 2008). Also, the part-whole concept that is necessary for understanding fractions, decimals and percentages is highly relevant in understanding rhythm (Rauscher, 2006). Révész (1954) asserted that the link between musical and mathematical skills stems from the ancient tradition of treating the study of music as a mathematical discipline and from the mathematical structure of music. However, the ability to create music may not require particularly high levels of mathematical thinking.

Another branch of research looks into creativity as common ground between two disciplines. We still know little about creativity in music and mathematics' relationship, even if creativity has been studied in the disciplinary perspective of mathematics and music (Sriraman, 2014; Odena, 2012; Webster, 2003).
In mathematics, the association between the richness of the problem (complex, open, ill-defined and contextualised) and the creativity of solutions (originality, flexibility and fluency) was highlighted by Manuel et al. (2012). For their part, Bélanger et al. (2014) have enriched the theoretical frameworks of the didactics of mathematics and creativity in which take place the concepts of inventiveness, imagination and situated cognition (Smolucha 1992).

In music, the authors postulate that many creative sounds are spontaneous in children (Kaddouch, 2006; Soulas, 2008). However, in a recognized creative field as music, the teaching of creativity is often achieved in normative frameworks and often left behind (de Champlain, 2010).

Through technology, music composition related to mathematics had remarkable growth over the past decades. Recently, the Fifth Biennial International Conference on Mathematics and Computation in Music (MCM2015) (22-25 June-2015, UK) brought together researchers from around the world who combine computational mathematics with music theory, music analysis, composition and performance. In our exploratory study, we will look at possible connections between technology and creativity through mathematical patterns created by young children using iPads.

4. Theoretical Framework

4.1 Cultural Framework of Creativity

Creativity is recognized as a tendency to make a production that is both new and in relation to the culture in which it occurs. It is described as absolute creativity (Levenson, 2011) if it makes a special contribution to society and it is described as relative creativity (Levav-Waynberg & Leikin, 2012) if it is based on the interpretation of the knowledge system of a person, which seems closer to the student activity. As a complex concept, we would define it in terms of culture and in terms of affordance.

We propose to adopt the *socio-cultural psychology* of creativity (Glaveanu, 2014) where a creative person is seen as a socio-cultural entity. Culture consists of a network of meanings, an inter-connected system of signs (Geertz, 1973) that stands the test of time (Jovchelovitch, 2007) being preserved and transmitted between generations. The creative acts are therefore considered in their nature and origin as socio-cultural. Therefore the individual cognition has a social origin and expression. As creativity and culture influence each other without dichotomy (culture exists around individuals and determines their conduct), creativity remains under the influence of culture and society (Glaveanu, 2014). With "culturally impregnated" materials, creativity leads to the production of artefacts that are valued as original.

The theory of affordance focuses on the relationship between the environment and the learner supported by "culturally impregnated" tools perceived as useful (but in different ways by different individuals) in a given situation (Gibson, 1977). "An affordance is a relationship between the properties of an object and the capabilities of the agent that determine just how the object could possibly be used" (Norman, 1988, p. 9). The concept of affordance applies to creativity both in music and mathematics: in solving mathematical problems and creating music, students are encouraged to share their knowledge, their imagination, their identity, their culture using different affordances, including technology.

4.2 Zoom on school curricula in terms of creativity

For Mathematics, the French curriculum of the Ministry of Education of New Brunswick (2012) recommends to "use mathematics in other subjects of the curriculum - arts, music ..." (p.20). For Patterns and Algebra, "students (3rd year) need to explore patterns: to create new sequences from non-numerical sequences with complex repeating pattern on independent attributes and different structures." (p.80). For Geometry: "Demonstrate an understanding of geometric shapes to create new ones. To represent geometric shapes: creating solid models using concrete materials." (p.82)

For Music, the French curriculum of the Ministry of Education of New Brunswick (2007) recommends to "create instrumental pieces from the different elements of musical language." (p.18)

There is a good reason to highlight the analogies between music and mathematics, and this relates to the field of education. In the Middle Ages, for instance, music was taught in the "Quadrivium", which included: Arithmetic, Geometry, Astronomy (Riché 2000).

This learning environment contributed to musico-mathematical discussions and reflections that led to major discoveries in music. In the 1500s, music schools were launched in Italy with the creation of "Conservatorio" (orphanages where children learned music by interpreting it in recognized cultural norms). It is legitimate to ask if the way we teach music in our schools is inherited from this vision of music education.

Now, Music and Mathematics are separate disciplines taught in our schools. We hypothesize that new links between mathematics and music can be recreated through interdisciplinary skills with ICT by focusing on creativity.

4.3 Creativity & Technology: new affordances and new practices

To create, students used Digital Audio Workstation Garage Band on iPad. For Savage (2005), it is a "Mega instrument" with a wide range of audio material easily manipulated by simple interface (Fig.3). It allows students to quickly generate, develop, implement and share ideas to create music (Gall and Breeze, 2008).

4.3.1 Description of the Digital Audio Workstation Garage Band:

This software allows the user to create music by playing different instruments displayed on a touch screen (Figure 1 and 2). When the user wants to check what he recorded, he can see that what he did is represented by colors in a box that varies in size depending on the length of his musical creation (Fig. 3).

Figure 1

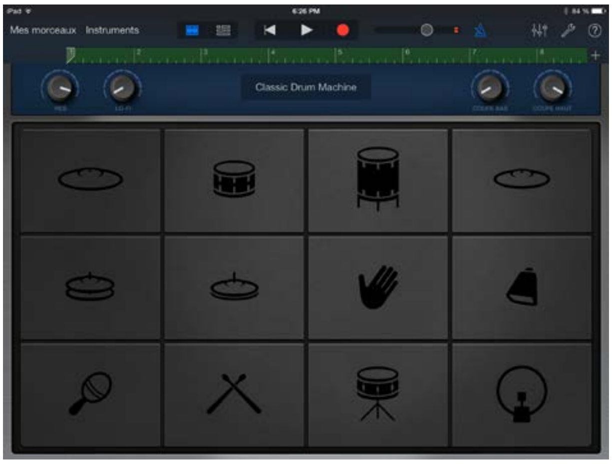

Figure 2

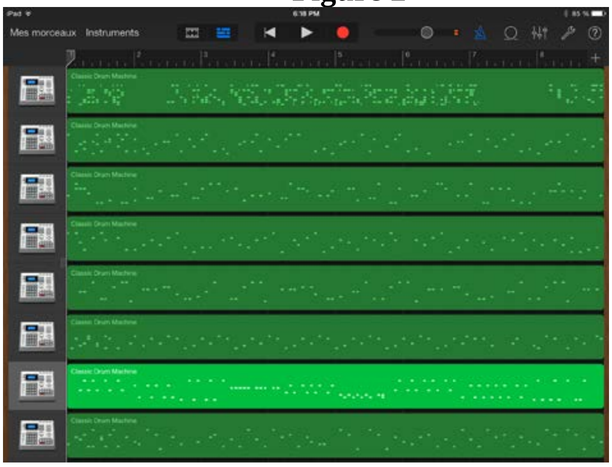

Figure 3

5. Methodology

The musical creative process suggested to students is inspired by Schaeffer's method (1966) to capture the potential of recorded sounds. They are listening effects (listening to the sound with no other purpose than to hear it better), listening changes (when sound forms are repetitive) and signal changes (opportunities are available to work on the sound). We believe this method is appropriate for understanding how students create without a proposed scenario defined in advance; the only option is the affordances of Digital Audio Work Station.

Our subjects were 6 third year students (8 years old) of a primary school in Moncton, New Brunswick. Chosen by the teacher based on the following criteria: Not necessarily taking music lessons outside school; Equal number of girls and boys.

There was 10 thirty minutes sessions of creation: students, grouped in one room, individually created music with an iPad - GarageBand and headset. Data collection was in form of observations, elicitation interviews (Vermersh, 1994) and recordings of works.

6. Results and interpretation: creative processes explained by students:

In this study, to achieve elicitation interviews (Vermersh, 1994), the researcher asked students to explain how they were creating.

6.1. Examples of mathematical reasoning speech of students who create music.

Example 1: To achieve his musical creation, the student used the acoustic guitar (Fig.1)

Student 1: “I played twice the same thing. This is where it stopped.” (Fig.4)

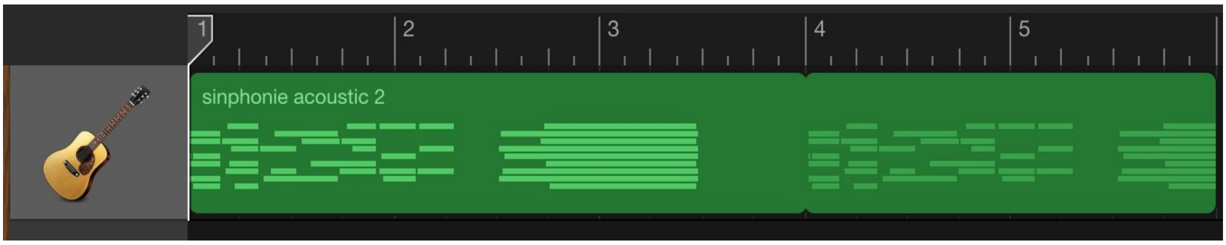

Figure 4

Example 2: In a subsequent work, this student developed this idea with several percussion instruments included in the classic drum machine (Fig.2).

Student 1: « … I played again to try to keep what I had done first ... For all this to go together … When it made the drums, I did it again when it made the cymbals, I did it again. »

The following key points emerged from the above student’s explanation: student 1 has created a pattern. According to the mathematical points of view of the curriculum, he used mathematics to create non-numerical patterns. In the second example, the same idea with several percussion instruments caused that non-numerical pattern changed from a percussion instrument to another. He also claimed that things go well together which is relevant to auditory sensation. Consciously or unconsciously, he made the link between mathematic and music and created non-numerical patterns in his musical creation.

Example 3: A second student used the « classic drum machine » and explain that: « There’s three rows and I pressed the instruments gradually …. when I was finished, I was re-doing it , I also was doing uh, I was clapping 1, 2, 1, 1, ... 1,2,1,2,1,1, 1 like that. » (Fig. 5)

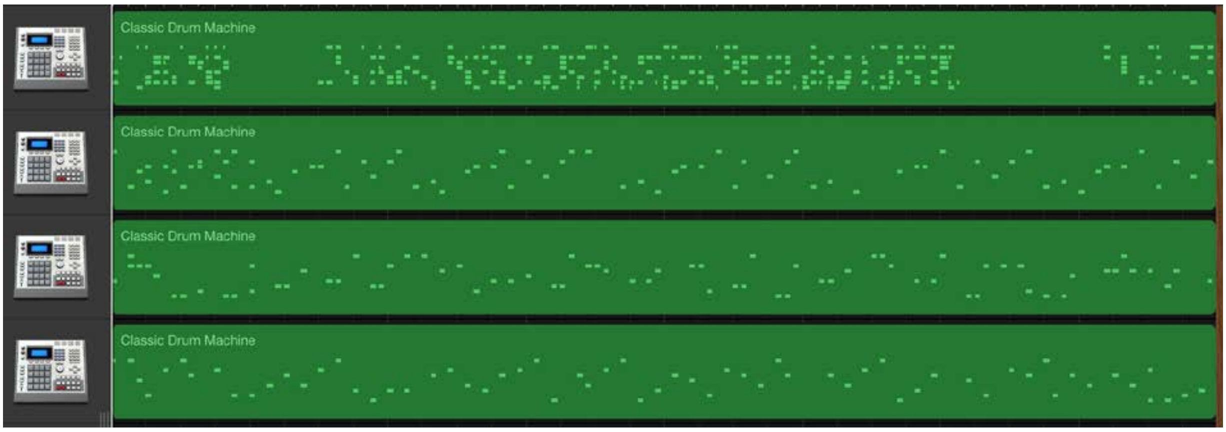

Figure 5

The key point here is that the student has created a much more complex pattern, which she verbally explained. According to her, this is not just something that is repeated to create her music; she juxtaposed sounds of various percussion instruments one over the other (this is called "polyrhythm" (Abromont, 2001) in music) instead of playing them one after another. When you listen to this creation, it gives the impression of a more intricate pattern.

In music, there is a method of musical units segmentations proposed by Ruwet (1972) based on the principle of repetition. In the explanation of students (and when we listen to their works), we realize that the way they conceive music is also based on this principle which is a mathematical reasoning. Furthermore, we can see that the idea that music comes from auditory sensations is present in the student speech (one student expressed concern that it go well together). Thus, the 2 currents, mathematics and feeling, interact. But some questions remain:

- Is it the affordances of the software that have created this mathematical perception of music (interfaces indeed present the music instruments geometrically)?
- Is the mathematical structure of the composition of the two students more dominant and more complex than the feeling?

7. Conclusion

To end this paper, we want to outline that being aware of the links between mathematics and music through creativity is a sure way to learn and understand one and the other, and maybe to learn and understand others fields too:
« Perhaps the most general aspect of the affinity between mathematic and music might be the perception and articulate study of patterns. Pursuing this agenda within music might encourage children to become intrigued with patterns in other domains as well » (Bamberger, 2013).

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THE CONNECTION OF MATHEMATICS AND MUSIC AS AN OPPORTUNITY TO CHANGE BELIEFS

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Abstract: *The strong connection of a teacher's beliefs and the beliefs a student gains when learning from this teacher is well discussed and has shown up in several studies. (Philipp 2007, TALIS 2009, Pajares 1992). I have always wondered why it is widely accepted that music is a subject, which you can learn and perform with delight, whereas mathematics often provokes a rather anxious feeling in learners as well as in many teachers. (Beilock 2009) In my article I present my idea to change misbeliefs about mathematics by using interdisciplinary content of mathematics and music.*

How to change beliefs

I became interested in a study conducted by Lynn Newton in 2013. She investigated the beliefs of 49 pre-service teachers and asked them to decide which subject in their opinion offers the opportunity for creative thinking on a scale from 0 to 12, with 0 meaning "no opportunity" to 12 representing the maximum opportunity.

> *"The major conclusion is that these teachers hold the general notion that the arts (subjects like music and art) are creative while other 'non-arts' (subjects like science, mathematics or ICT) are not." (Newton 2013, p.37)*

In fact, the average result of music was about 9 and mathematics got barely 1.
Although these results are quite disturbing they triggered some ideas that I decided to investigate my own studies.
For mathematics teachers, it is important to be aware of the fact that their own beliefs can influence the beliefs of their students regarding mathematics. For instance, teachers who think, that mathematics is a subject without the opportunity of being creative will spread this misbelief. (Beilock 2009)

> *"Beliefs are unlikely to be replaced unless they prove unsatisfactory, and they are unlikely to prove unsatisfactory unless they are challenged and one is unable to assimilate them into existing conceptions. […] A number of conditions must exist before students find anomalies uncomfortable enough to accommodate the conflicting information." (Pajares 1992, p.321)*

Hence, according to Pajares, challenging those (mis-)beliefs by providing students with conflicting information, could eventually help them change their beliefs.
Fortunately, this is exactly what Lynn Newton found - a big field of conflict between the beliefs her pre-service teachers have concerning mathematics and music. So I started to think that the combination of these two subjects might offer us the opportunity to help changing students' beliefs. More specifically, by exploring students' experiences with music, we could try to generate conflicting information which would have an impact on their beliefs.

So I started analyzing different sources to find out more about the connection of mathematics and music on a historic perspective. After that, I had to study the beliefs of students to see if there is conflicting information for them and finally I had to develop ideas with connections to use in class.

Historical sources

In the ancient history, we can find several examples of famous personalities who have dealt with both subjects.

For example, Pythagoras who held holistic views of the subject of his thought, is known as a mathematician who was also a founder of theoretical thoughts about music. Even if the existence of Pythagoras as a real person is still questionable (Critchley 2008, p.7), we know that all the things we can read from Pythagoreans show this distinct holistic view onto everything they were concerned with. (ibid.)

Other well-known personalities in this context are Plato, Aristotle and Socrates. Although they were neither mathematicians nor musicians they gave us a clue on how they thought about these and other subjects. To them, one should be concerned with 'beautiful things' and it is beauty they found in mathematics as well as in music and other disciplines. (ibid. p.19)

From the classical antiquity to the middle age, we find a first division of essential scientific subjects into the "septem artes liberales" – the seven liberal arts (grammar, logic, rhetoric, arithmetic, geometry, the theory of music, and astronomy). But although they were divided, they still were connected with the "queen" – philosophy. (Conway 1959, p.8)

For Gottfried Wilhelm Leibniz, music was *"the pleasure the human mind experiences from counting without being aware that it is counting."*
(Berlin-BB. Akademie der Wissenschaften, p.158)

> *"May not music be described as the mathematics of the sense, mathematics as music of the reason?"* (J.J. Sylvester cited in Dieudonné 1998, p.VI)

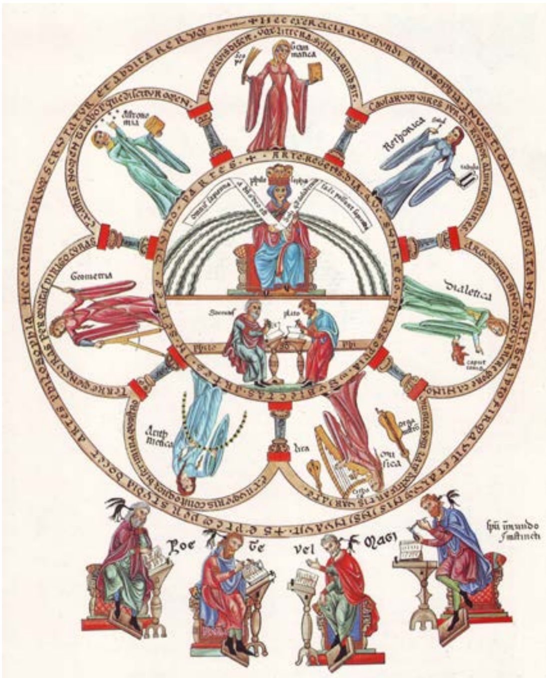

Figure 1. The seven liberal arts and philosophy (photography by Dnalor_01, Wikimedia, license CC-BY-SA 3.0)

Throughout the whole history, we can find a great number of similar quotes pointing to the existence of a strong connection between music and mathematics. Nevertheless, we experience a big discrepancy in the beliefs concerning the two topics nowadays. In order to get a deeper insight into this phenomenon, I conducted a qualitative study which focuses on student's beliefs.

Qualitative study

In order to evaluate, whether students' views concerning mathematics and music places them in an adequate conflict with their beliefs, I conducted a qualitative study. First I tried to find the most appropriate way of data collection, considering that "[…] researchers make various attributions to teachers through choices about data collection, theory, analysis of data, and presentation of findings." (Speer, 2005, p.361)

I chose Mayring's (1996) method of group discussion which offered the best insight into students' beliefs.
Following Mayring's advise about how to form groups, I tried to build what he calls 'natural groups', meaning the interviewees should know each other before the discussion and have some natural connections to the group in their everyday life.

I was able to form three natural groups. The first two groups consisted of 14 students who studied in one of my classes. As I wanted the group to be the most natural as possible. I looked out for similar subjects they were studying. Coincidentally 7 of them were students of mathematics and music, so I decided to put them into the first group. The second group therefore consisted of 7 students studying mathematics and not music. I asked students from this group to participate in my study since I noticed that they tend to always sit next to each other or even spend some time together outside of my class. The third group was formed by 8 university teachers. The group members used to meet every week, which also made them being a part of a 'natural group'.
According to Mayring, a group discussion should not be too pre-structured from the researcher's point of view, in order to allow the interviewees, unfold their attitude as unbiased as possible. Also the researcher should not interfere as long as it's not inevitable.
I asked the groups to discuss the topic "The connection of mathematics and music". By choosing this topic, I was planning to look out for beliefs they would appear from what the participants were saying and how they wert saying it, as suggested by Nentwig-Gesemann (2001). Also, in order to comply with Mayring's suggestions, I did not let them know that beliefs were my real interest.

I videotaped the discussions and transcribed them, then analyzed the data using the Documentary Method (Nohl 2006, Nentwig-Gesemann 2001). My focus was to uncover the students'' beliefs, so I chose to transcribe not only the spoken words, but also non-verbal behavior while looking at all possible ways of showing emotions. After that I classified data into different types of behavior and looked out for similar patterns and other relevant incidents.

My findings

Overall, the discussions took about two hours. The first two groups discussed for about 30 minutes each, the university teachers for 60 minutes.

The first group found their way into a discussion quite easily. I asked them about the connection of the two subjects and did not interact with the group right from the beginning.
The second discussion went quite differently. The students had no idea about any connections. So I tried to find other topics to generate text, which could help me find out about their beliefs.
The university teachers made it easy for me as they discussed the topic from a very rich diversity of angles.

Even if each group approached the topic in different ways, all three groups showed similar patterns regarding their beliefs. The sentences I could link the most to a positive belief towards mathematics were the following:

- (I,A) : *"In school I preferred maths."*
  "For me, music and maths are a passion."
- (I,C): *"I like to say: 'I'm studying mathematics.'"*
- (III,F): *"Let me try to make you like maths."*
- (III,H): *"I found it sort of beautiful to reduce fractions."*

[(I,A) means: group one, participant A]

Most of the other statements were showing rather rejection, anxiety or even anger towards mathematics. The students of both groups were a bit reserved, but nevertheless clear with their opinions.

"In school, maths teachers often cater their lessons to the gifted kids. I often could not follow." (I,B)

"My friend is ***really*** *studying mathematics at the university of Munich. Everyone goes like: 'Wow! Crack!'"* (I,D) (Compared to him/her "just" studying didactics of mathematics.)

Three of the university teachers repeatedly showed their anxiety by introductory sentences like:

"Well, I am not a real mathematician, but […]" (III,D) - *"I'm not into higher maths."* (III,F)
"I'm no mathematician too. (ALL laugh.)" (III,E)

These sentences were spoken very apologetically.

Another quote, *"It's no wonder students have problems with maths. It's because of the way they are treated."* (III,C), expresses a very honest negative comment in the same group.

The beliefs concerning music were rather in direct opposition to ones about mathematics. They can be illustrated with following quotes:

"I love music." (I,B) – *"In school I always liked music."* (I,G) – *"I always preferred music. Well, playing an instrument."* (I,A)

An important finding to me is, that in every group, I found a short discussion about "one mathematics teacher" being sort of different to others. A personality, which enabled the respondent to develop a new belief or change existing beliefs concerning mathematics.

While the findings from this study seem to comply with the ones found in the literature, they allowed me to elaborate some teaching ideas to bring up the conflict that would challenge negative beliefs about mathematics in school or in teacher education. These ideas can be used to show the beauty of mathematics in its connection to music and eventually help learners to think about mathematics and music in a new way.

Beauty you can hear

The experiments of Pythagoras on a monochord are a well-known connection of mathematics and music. A string divided into different proportions produces different musical tones.

C	C#	D	D#	E	F	F#	G	G#	A	A#	B	C’
$\frac{1}{1}$	$\frac{243}{256}$	$\frac{8}{9}$	$\frac{27}{32}$	$\frac{64}{81}$	$\frac{3}{4}$	$\frac{512}{729}$	$\frac{2}{3}$	$\frac{81}{128}$	$\frac{16}{27}$	$\frac{9}{16}$	$\frac{128}{243}$	$\frac{1}{2}$

Playing two of these tones simultaneously gives you a musical interval. Although this connection is very popular, we can make a better use of this direct connection between music and mathematics in class.

When you are hearing an eighth, you in fact hear the fraction $\frac{1}{2}$. When you hear a fifth this is the sound of perfect $\frac{2}{3}$.

If we now dare to talk about beautiful and less beautiful sounding intervals most people judge a perfect eighth, fifth or forth sounding very nice, whereas a minor second or a triton sounds less harmonic. Seeing that these intervals are representing the fractions $\frac{243}{256}$(minor second) and $\frac{512}{729}$ (triton) gave me the idea to use this connection to talk about fractions and intervals in a new way.

While seeing a big potential for the didactics of both subjects, I argue that such examples can bring upfront a very new thing, less familiar to pupils or students - the sound of fractions. We can use this in school in many ways. For example, we could do experiments with monochords and listen to the sound of different fractions. On the other hand, we can find a way to introduce intervals in a music class and work interdisciplinary together with a mathematics colleague.

Beauty you can see

In his book „A Geometry of music”Dmitri Tymoczko gives us other ideas to see and therefore talk about music in a new way. He visualizes music by giving it a geometric shape.

Although the circle is already used in music in the circle of fifth, shaping a simple scale that way enables us to see single tones as well as harmonics in a mathematical way and interpret it inner-mathematically.

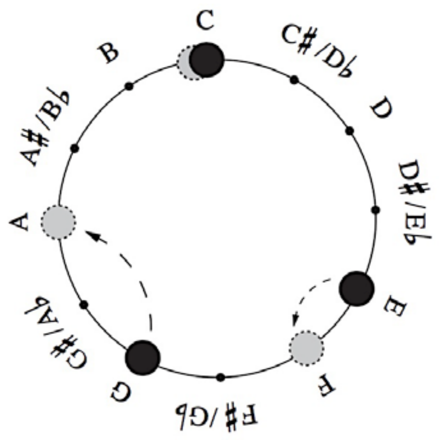

Figure 2. Dimitri Tymoczko´s shape of a scale (Tymoczko 2011 p. 37) Used with the authors permission

The illustration 1 for example shows us the key C major transformed to F major. As you can see you can reach the F major chord in at least two ways. You either move the two tones E and G to F and A (as illustrated here), or you can transform the three tones in one rotation from C, E and G to F, A and C.

Another way Tymoczko uses this is to watch keys whilst listening to a piece of music. That way every key of the piece gets an illustration and is transformed to one another. On the first sight it might look random but the inherent structure of the musical keys fits the audible and visual sensation in a way that is new to me.

This circle is just one new shape Tymoczko developed in his work. He transferred this idea, which shaped the scales in two dimensions to three and even to multidimensional spaces.
In my opinion this “chord geometry” as he calls it is not only a new view onto music but also another link between music and maths and should therefore find its way into classes.

Beauty in the language I

The language of mathematics and the language of music are compatible, as I will show you in my last two examples.

There actually is sound in the following mathematical expression:

$$H_n = \sum_{k=1}^{n} \frac{1}{k}, n \to \infty$$

The harmonic series implies the sound of a vibrating violin string, as you can read in Marcus du Sautoy’s “THE MUSIC OF THE PRIMES”.

> *“Since the sound of a vibrating violin string is the infinite sum of the fundamental note and all the possible harmonics, mathematicians became intrigued by the mathematical analogue. The infinite sum 1 + ½ + 1/3 + ¼ ... became known as the harmonic series.” (du Sautoy, 2004, p.79)*

He shows us another possibility to talk about music in a mathematical way. Not only the mathematical expression „harmonic series“ is used in music already (it’s also called overtone series) in my opinion this fact can be used to try to find different mathematical expressions for the overtones of different instruments. To think about a more advanced way to use this, we can start to imagine how mathematical rows sound like. Maybe that seems strange at first. But it's worth an experiment.

Beauty in the language II

As Rudolf Wille shows us in his Definition of a pitch space, even the way of mathematical definitions can be used to describe musical phenomena:

- A ***pitch space*** is an ordered pair (T,h), where T is a set and h is an injective mapping of T into the set R+ of all positive numbers. The elements of T are called ***tones***, and for $t \in T$ h(t) is called the ***pitch***.
  (Wille, 1976, p. 239, translated by the author)

Using this definition, you now can create a pitch space. For example, you can define an equal temperament by choosing the following set and injective mapping:

$$T := \{-48, -47, ..., 35, 36\}, h(t) := 440 \times 2^{\frac{t}{12}}$$

If you pick the 0, which is an element of T, the result of the mapping h(t) is 440. In Hz it gives you the tone a. All the other numbers of the set will give you all the other tones on a piano tuned in equal temperament.

We can use this fact in teaching. For example, we can experiment with different injective mappings to generate different pitch spaces. I think of a program that directly lets you hear the pitch space you get when choosing different mappings. So this definition can, on the one hand, be used to understand the way mathematical definitions are formulated, and on the other hand, to learn more about pitch spaces.

Conclusion and perspectives

Mathematics and music are strongly connected. In the article, I used a historical basis which allows for pointing out some connecting ideas. The starting point of my research was the gap between the beliefs people hold about the two subjects, which triggered the idea to use this kind of tension to influence beliefs. While this idea of confronting two opposite views is quite a new in education, the data from qualities study with students and faculty show potentially promising paths for research and practice.

One of such paths would be to develop and investigate more relevant interdisciplinary content and investigate its effect on the beliefs. Some well-known sources (e.g. Wille 1976, mono-chord experiments) and brand new concepts (Tymoczko) did not find their way into classes yet. I suggest to re-exam their educational value within the outlined context.

I was professionally trained in mathematics and in music, therefore my thinking of these two subjects was connected quite early in my life. Actually, to me these two subjects are (more or less, I admit) part of the same science. In mathematics we search for proofs of our theorems which according to Coxeter (1962, pp.13-24) has a lot to do with the creative process of composing music. He also used to call both subjects "arts". I strongly support that. Mathematics investigates structures (algebraic structures as groups and fields, measures, equivalence relations, metric structures etc.) as well as music does (structure of tones, harmonics, melodies, formal structure etc.). To be a good or even professional musician you have to do a lot of (finger) exercise. Maybe these thoughts can help to overcome the commonly misbelief, that mathematics is a subject that you either are good in, or just can't succeed. Ask any mathematician and you will be told that exercise is a fundamental part of mathematics education.

Although these similarities seem so obvious, I showed that the perception of my interviewees is quite different. My research reveals that fact, but does not answer the core question of whether it is possible to change beliefs in that way?

Another opportunity for research is the question about the qualities of "the one teacher" to whom I find at least one reference that in each group. Namely, there was at least one person telling that they have met a teacher, who gave them the opportunity to develop a new insight to the field of mathematics.

When I presented this to colleagues and students I got the feedback, that they agree with me that this actually could help change beliefs. Although this is very encouraging to me to follow that path, I'm still concerned. My presented findings about the beliefs did not really disturb anybody in my audience. It seems like it's a common and accepted fact, that a lot of people could be either anxious or reluctant, or even angry when it comes to mathematics.
A real issue to me is that university teachers also showed some of these negative feelings when talking about maths in their group discussions. Surprisingly, they are teaching teachers, so they are, sort of, on top of a big "chain-reaction".

Facing this, I come to the conclusion that we need a lot of research and new ideas to enable people to see the beauty of mathematics.

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MATHEMATICS IN CONTEMPORARY ART

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Abstract: *The Niebra-disk (Germany, plane mapping of the three-dimensional sky), imprints in Central American caves (number representation by fingers and hands, Egyptian temples (rotational symmetric arrangement of four stairs leading into the holy pond at the temple of Denera), ancient mosaics (Spain, Alhambra), Pythagorean harmonics, Fibonacci numbers, golden section and perspective mapping show: mathematical methods help artists in creative processes since the beginning of time.*

For some mathematical topics the lecture presents modern examples of their application in art like: mirroring, patterns and numbers, combinatorial analysis, hazard, parallels in mathematics/science and art, computer science, "prescientific mathematics" and concrete art, aesthetics of information, falsification. The lecture refers to exhibitions curated by the author in Duisburg (DMV), Potsdam (GDM), Sevilla (ICME), Berlin (ICM) (for illustrations please look at their cataloques, too).Also we show few possibilities of interdisciplinary teaching in regular schools and for special groups (immigration background, lower cultural level, …). In the end we dare a view into the future of the cooperation between mathematics and contemporary art, which might be possible.

Introduction

In order to have a common starting point, we declare some basic statements (no premises in mathematical sense) following to different encyclopedias thereby:
ART is the name for everything, which is created by men and has only one or no function.
ART does not abrogate earlier artworks.
If something is ART or not, depends of the epoc.
MATHEMATICS researches patterns and relationships between objects (digits, variables, …) of our thinking.

Mathematics as a tool

At first we will show, that mathematical/geometrical methods sometimes were used by artists, even there where art-historians believed to be sure, that these artists positioned their objects within their artworks only led by their personal feeling.

For this let us have a look at a picture of Kasimir Malewitsch.. We found out for this picture as well as for several others made by Malewitsch during that time: Malewitsch constructed these pictures. He did not set elements only by his aesthetical feeling into these pictures. By the way: we found this fact for instance in artworks of Moholy-Nagys realized ten years later, too.

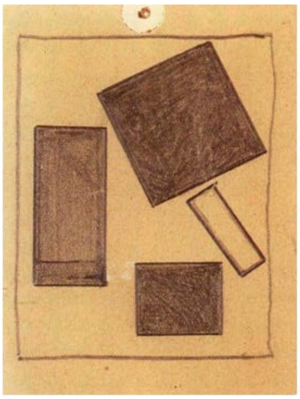
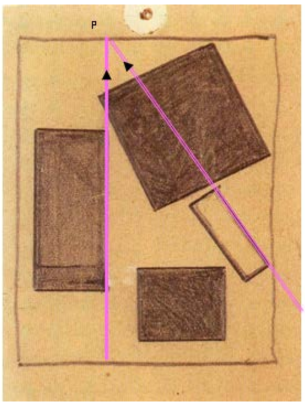
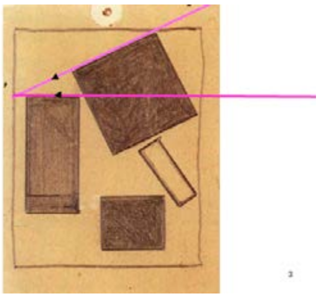
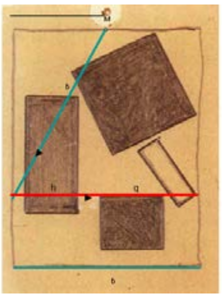
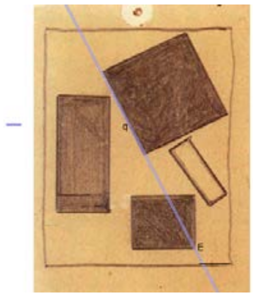
A.  B. 

Figure 1 and 2
A. Kasimir Malewitsch, “without.title”, 1915, pencil on paper, 8,8 x 6,5 cm , published in Museum Ritter: SQUARE - Die Sammlung Marli Hoppe–Ritter, 2005 – 2006, p. 205 (cataloque) *)
B. Connection between one side of a black rectangle, one side of the white rectangle and a point P on the upper side of the picture.

C D E

Figure 3, 4 and 5
C. Connection between one side of a black rectangle and one side of another black rectangle and a point P on the left side of the picture.
D. Connection between the point in the middle of the upper side, the width b of the picture, the not used line h and the upper side q of a black square.
E. Connection between a side q of a black rectangle and the corner E of the third black rectangle by a straight line..

Let us recapitulate: For the Construction of the content of his picture Malewitsch used points, segments and lines which are given by the shape of the picture before as for instance the middle point of the upper side, width of the picture. This fact leeds to an important difference to Max Bill, Camille Graeser and some other members of the famous group of Zürcher Konkrete coming up in the late thirties of the twentieth century as we shall show later in this lecture.

Calculating Beauty (Historical Trials)

After one example in general of using mathematics as a tool in art we brought some remarks on historical trials of how to calculate beauty. First we showed a model of a greek statue in connection with the Canon of Polyklet. Polyklet (about 460 to 420 before Chr.) was a greek artist, who created a “canon” of human proportions. He defined for instance that the knee of a person has to be positioned at the height of 4/15 of the total height. For more than one century - nearly every Greek sculptor used this Canon of Polyklet.

Lateron the School of Pythagoras defined that integer relationships guarantee harmony. To demonstrate this with an – more humorous - example, the photo of a famous German female model from nowadays was shown, whose body had the relationships 90:60:90. Thus even members of the School of Pythagoras would have called this a harmonic proportioned body!

"Prescientific" Mathematics

Also today artists use so called "prescientific" elements of mathematics such as plane mappings, straight lines and their parts, elementary plane and spatial forms, elemtary topology, numbers, series of numbers, elementary operations like addition and subtraction and others.

At first the plane mappings mirroring., translation, enlargement und rotation in contemporary art were shown by a few examples:
Starting with the translation the first example was a picture of an oversized bureau in China with hundreds of workers and the second was a photo of two more than hundred meter high parallel lights during a ceremony for the destroyed World Trade Center in New York. In the end a picture of the Spanish artist J. Genoves showed, that mathematics – here geometry – can demonstrate political incorrectness for instance with a group of three walking men, two policemen and a client in their middle. The artist doubled this group by translation thus that one group followed the forgoing one. And in the same way triples of men came from the opposite side, too. That meant in the time of making this artwork: the systems , especially in South America changed; but the bad methods rested the same ones.

During the Biennale in Venice in 2015 a very special aspect of mirroring existed with the installation "My East is Your West" , were two artists, Rashid Rana and Shilpa Gupta. from India and from Pakistan in their artworks reflected their different view on their common border.

Günther Uecker„Sandspirale",1970, for the artist a tool for his meditation, shown in the Martin-Gropius-Bau, Berlin in June 2015 and Isa Genzkens "blue, green, yellow ellipsoid 'JOMA' " presented in the MOMA in NY in 2014, both in the exhibition "mathematics in the art of the recent thirty years" in Ludwigshafen 1987, too, illustrated the respect of modern artists for the impression and the beauty of mathematical forms.

Here some remarks and pictures concerning special auditors in mathematical education were added: In order to increase the self-awareness of children with immigrate background it could be helpful to demonstrate in the classroom, that in their former homeland high culture existed, too: Series of puppets out of former soviet regions include spatial enlargement. African clothes often show astonishing examples of translation of motifs. Islamic mosaics in Iraq, Syria and other Arabic countries include every type of tessellation. Often pupils not talented for mathematics astonishingly like to cooperate in that moment, when the results of their working in mathematics appear as artworks (tessellation, axial symmetry, translation, combining of coulered cubes in a row,…)

If something is ART or not, depends of the Epoc

Our next subject was initialized by a show in the Pinakothek der Moderne in München in 2015-2016. Hitlers favorite artist Adolf Ziegler„Die vier Elemente“-„the four elements“- before 1937 and an artwork of Max Beckmann „Versuchung“-temptation“, 1936/37
were used in the lecture to test the for mathematics false but for art correct seeming declaration from the beginning of the lecture: “If something is ART or not, depends of the epoc”. During the Drittes Reich (1933-1945) a poster of Zieglers piece nearly hang in every house. After the end of Second World War it totally vanished, since it now only for comparison to in that time dammed artworks of Max Beckmann and others got presented.
Concerning Beckmann: In February 1938 the Kunsthalle of the Swiss city Bern showed pieces of Max Beckmann within a group-exhibition brought over from the Berlin show on “degenerative art”.

Reaction in the Swiss Press: The Swiss Berner Tagblatt (free translated by the lecturer): „(This art) bears something like diseasedness, perversity thus, that one only is able to approach with distaste for this degeneration („Entartung“).“
The Swiss Newspaper „Front“ in Zürich published a letter: „Lots of (pseudo-serious, lecturer) Critics („Kunstverdreher“) , with names ominously seeming jewish, find interpretations beyond belief for the art of Max Beckmann … We leave the Kunsthalle with the feeling, that the mopping-up operation within German art is not a shame but a good deed for the German culture.“(quoted: F.Lerch, s. below))

Today, in 2015 the Swiss Kunstmuseum Bern is fighting to get Gurlitts collection of these formerly „degenerate“ called art as heritage including works of Max Beckmann .The author even saw a well known Galerist from Bern begging for that openly during an public event.[1)6)]

Concrete Art

Concrete Art may be seen as one of the closest connections between art and mathematics. Max Bill as a member of the group of Zürcher Konkrete defined in 1938: “Concrete Art is created by the human mind for the human mind“.
Only one example may show some of the principles which concrete artists use:

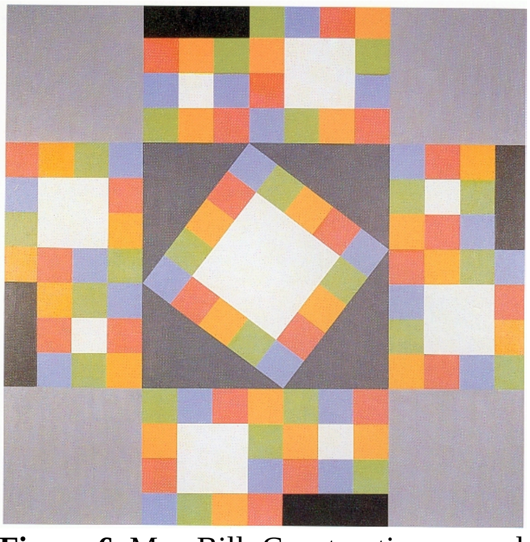


Figure 6. Max Bill: Construction around the subject 3:4:5
(Konstruktion um das Thema 3-4-5), 1980

- Quadratic format of the picture – the mostly neutral shape beside the circle
- Picture constructed with the help of a modul (here a small little square) – following modern industrial developments
- Rotation symmetry of the picture – here only in spite of the shapes
- Cyclical exchange of colours around each square
- Equality of the surface areas of the colours – for some concrete artists like Richard Paul Lohse a symbol for democratic appearance of all (colours)

All these principles are based on the human mind in complete difference to the artworks of Kasimir Malewitsch, where .all points, lines, corners,.. are given by the shape itself.[2)3)]

Cooperation Art - Mathematics

The last part of the lecture contained very different kinds of cooperation:

- A poster developed by the author for the DOCUMENTA IX in Kassel (1992) explained to the organizers of the famous exhibition (Jan Hoet, his team und artists being involved) how their keyword “displacement” for the coming DOCUMENTA IX might be declared members of the press and to the public by mathematical visualisation.
- - The mathematician Thom and the artist Salvador Dali developed together the theory of catastrophes within long discussions. Dalis “catastrophical” picture of a face, a flower and a dog changing nearly by chance from one to the other appearance illustrated this part: (Salvador Dali*„appearance of face and fruit bowl at a beach“ published in* Staatsgalerie Stuttgart, Kunsthaus Zürich: Salvador Dali, Verlag Gerd Hatje Stuttgart 1989, page 255) By the way: Dalis last picture was a drawing of one of Thoms elementary catastrophs.
- In 1973, more than ten years before .Daniel Shechtman (He later won the Nobel prize for this in 2011) found the base for the existence of quasicristals the artist Gerard Caris from Utrecht (NL) constructed such things as artworks (today to be seen in several universities in the Netherlands, in the ZKM in Karlsruhe and in the Museum Kulturspeicher in Würzburg).[4)5)]

Finish and Outlook

The lecture ended with examples of artworks dealing with the infinity (Roman Opalka, Rune Mields, Karl Gerstner), with hazard (the glass window of Gerhart Richter in the Cathedral of Köln, Francois Morellet), with combinatorics (Peter Sandfort, the poem “Ireland” of the Concrete Poet Eugen Gomringer), dynamic Processes (Maschinenlabor of the ETH Zürich).

The future surely will bring closer contacts between art and mathematics/technics because of the new contents like codification theory, dynamic processes, virtual reality, augmented reality but also because of the velocity of the information streaming today between the disciplines.

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